

Student academic performance stochastic simulator based on the Monte Carlo method

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A B S T R A C T

In this paper, a computer-based tool is developed to analyze student performance along a given curriculum. The proposed software makes use of historical data to compute passing/failing probabilities and simulates future student academic performance based on stochastic programming methods (Monte Carlo) according to the specific university regulations.

This allows to compute the academic performance rates for the specific subjects of the curriculum for each semester, as well as the overall rates (the set of subjects in the semester), which are the efficiency rate and the success rate. Additionally, we compute the rates for the Bachelors degree, which are the graduation rate measured as the percentage of students who finish as scheduled or taking an extra year and the efficiency rate (measured as the percentage of credits of the curriculum with respect to the credits really taken). In Spain, these metrics have been defined by the National Quality Evaluation and Accreditation Agency (ANECA).

Moreover, the sensitivity of the performance metrics to some of the parameters of the simulator is analyzed using statistical tools (Design of Experiments). The simulator has been adapted to the curriculum characteristics of the Bachelor in Engineering Technologies at the Technical University of Madrid (UPM).

Keywords:

Stochastic simulator

Monte Carlo method

Student academic performance

1. Introduction

1.1. Motivation, aim and contribution

The Bologna framework established in Spanish universities has promoted new requirements to the new curricula. In particular, some specific performance indicators have been defined by the ANECA (National Quality Evaluation and Accreditation Agency), while others are specific of the university. Namely, academic performance indicators could be referred to the subject, to the semester, the academic year and globally to the degree.

For any specific degree, ANECA has defined two metrics as performance indicators: graduation rate (measured as the percentage of students who finish as scheduled or taking an extra year) and efficiency rate (measured as the percentage of credits of the curriculum with respect to the credits really taken). It could be relatively easy to assess the results for a specific subject, to identify strong and weak points, and to implement some educational changes to improve results. However, considering not only the significant number of subjects in a degree but also the number of tracks, this analysis is not straightforward. For this reason, simulation is the tool to achieve an overall insight into how successful the degree is going to be as measured by the performance indicators.

The purpose of this paper is the development of a stochastic simulator of student performance along a given curriculum, in this case, Bachelor in Engineering Technologies, in the Technical University of Madrid.

By modeling the passing or failing of subjects as events which occur with given probabilities, a series of performance indicators for the degree (curriculum) are estimated after simulation of cohorts of students along the full curriculum. This allows for evaluating the effect on

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changes in the indicators, resulting from modifications in pass/fail probabilities of subjects or any other parameters of the curriculum such as the maximum number of credits to be enrolled in by the student which depends of the specific university regulations.

Monte Carlo (stochastic) simulation is used in science and technology in two directions: first, to understand the behavior of stochastic systems and second, to estimate parameters of the system when analytical techniques are unfeasible. Here we make use of both: graphics of randomly generated student evolution is useful to understand the process, and analytical computation cannot be performed straightforwardly.

In this work, we have used the Matlab software to write our simulator, which includes interactive dynamic graphical displays providing an instant visual feedback with a series of output graphics for a better understanding of the results. Besides estimating the performance indicators for initial (nominal) values of the input parameters, a sensitivity study is also carried out to find out which inputs are more relevant, either marginally or jointly, to determine the output.

The developed software is generic, and can be straightforwardly adapted to any academic degree. Additionally, the number and/or definition of performance indicators can be modified by the user if required.

1.2. Literature review

There are many interesting papers focused on academic performance indicators of individual students for specific subjects (i.e., [Ghamdi, Bassioni, Mustafa, & Al-Hamadi, 2013](#); [Huang & Fang, 2013](#); [Zuviria, Mary, & Kuppammal, 2012](#)), or even for a complete year ([Ting, 2001](#)). In some works ([Farooq, Chaudhry, Shafiq, & Berhanu, 2011](#); [Maris & Jacobs, 1995](#); [Rodríguez, 2005](#)), several factors are compared and analyzed to improve the academic performance of students.

There are also some interesting references where simulation tools are used to create a specific curricula planning in order to improve student performance ([Dorca, Lima, Fernandes, & Lopes, 2012a, 2012b](#); [Ganjeizadeh, Motavalli, Leung, & Cruz, 2008](#); [Letouze, Ronzani, & Oliveira, 2011](#); [Perkins & Paschke, 1973](#); [Saxena & Singh, 2012](#)). A pioneering work is [Perkins and Paschke \(1973\)](#), which propose a simulation model to estimate the enrollments associated with various scenarios for the post-secondary education system of a state. The work by [Letouze et al. \(2011\)](#) is concerned with the development of software as auxiliary tool for master's degree project supervision management; [Dorca et al., \(2012a, 2012b\)](#) design a student performance probabilistic model, in the context of the so-called student's performance simulation process (SPSP); [Ganjeizadeh et al. \(2008\)](#) is concerned with simulation-based optimization of academic resources.

However, there is not, as far as the authors know, literature where simulation tools have been used to provide help in the design of new curricula, to predict their performance indicators and to identify, by means of a sensitivity analysis, those features which could influence the indicators. For example, as will be commented in the final results, one of the parameters with the highest effect of the graduation rate is the maximum number of credits for which a student may register along an academic year. By slightly modifying this parameter in the right direction, the performance indicators improve. This can only be checked out by running simulations, otherwise one would have to undertake very cumbersome computations or wait four years to observe the results. The tool presented in this paper is thus very useful when designing curricula and as a “workshop” to test new scenarios, based on the Monte Carlo method.

The Monte Carlo method ([Rubinstein, 1981](#)), originally developed in the neutron physics framework ([Metropolis, Rosenbluth, Rosenbluth, Teller, & Teller, 1953](#)), has been applied to a wide range of problems in Mathematics and Sciences. For instance, in [Mira, Martínez, and González \(2003\)](#), a stochastic simulator is proposed for an educational-tool of chemistry based on the Monte Carlo method; and the work by [Boucher, Bramoull, Djebbari, and Fortin \(2012\)](#) applies the Monte Carlo algorithm to analyze the student achievement. Finally, [Kleijnen \(2004\)](#) provides a review of the use of experimental design for sensitivity analysis to identify those parameters with highest influence over the performance indicators.

1.3. Paper structure

The rest of this paper is organized as follows. Section 2 describes the stochastic simulator developed, detailing the input parameters, the performance metrics, and the Monte Carlo method employed. Section 3 provides a sensitivity analysis for the performance indices considered. Finally, Section 4 provides some relevant conclusions.

2. Stochastic simulator

The proposed stochastic simulator is a computer-based tool which allows for simulating the student-curricula evolution for an academic degree. The software internally makes use of the Monte Carlo method to simulate the path of each individual, based on the passing probabilities of the academic degree implemented.

The simulator is general and it can be applied to any academic degree. In this paper, the degree is Bachelor in Engineering Technologies ([Study Plan, 2007](#)) in the Technical University of Madrid (UPM), Madrid, Spain.

The structure of the degree is the following (see [Fig. 1](#)): it contains 240 credits (ECTS) divided in four academic years. Each academic year is divided in two semesters. In each semester, the student is enrolled in one or more subjects, with a maximum of to 36 credits/semester. Each academic year, the student has three opportunities to pass a subject. In case of failing, the student can enroll again in the next semester. There is no restriction on the number of attempts.

At the beginning of the sixth semester, the students have to choose one of the eight following *majors* or *tracks*: Automatic Control and Electronics, Electrical Engineering, Construction Engineering, Mechanical Engineering, Materials, Industrial Engineering, Industrial Chemistry, Environmental Engineering and Energy Technology.

The aforementioned degree structure is implemented in the software routines developed. The next subsection describes the input parameters that have to be estimated and fed to the simulator.

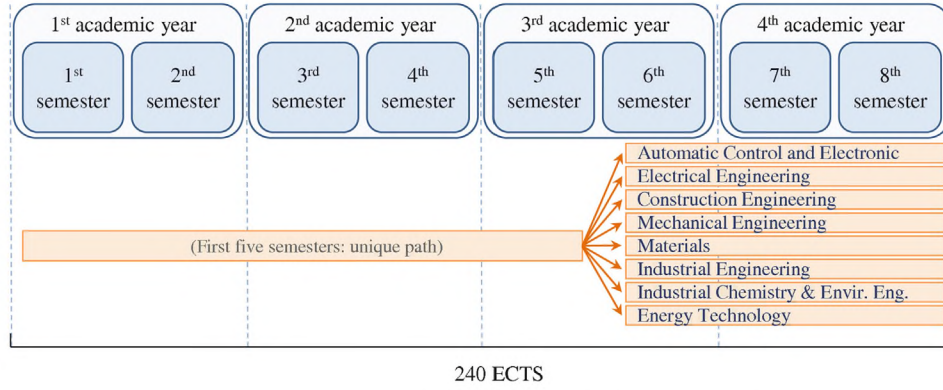


Fig. 1. Curricula structure of the Engineering Technologies at the UPM.

2.1. Input parameters

2.1.1. Passing probability

Each subject has a passing probability p_i , where subindex i indicates the subject number. This probability is estimated as the ratio between the number of students who passed the exam and the number of students who actually took it, using historic data (ETSII-UPM, 2011).

2.1.2. Number and distribution of students

As for the amount of degree students, two parameters have to be adjusted: (i) the total number of freshmen at the first semester, and (ii) the percentage of students enrolled in each major. Historic data is used to adjust these values.

2.1.3. Maximum semester credits

According to the academic regulations, there is usually a maximum number of credits that each student can enroll in for each semester. For the academic degree considered in this paper, this value corresponds to 36 credits.

2.1.4. Increasing passing probability for failed courses

Analyzing historic academic data it has been observed that, for a given student, if he/she fails a given subject, the probability of passing it in the next attempt is usually higher. Thus, parameter Δp is defined as the average increment of passing probability for failed subjects. The value of this parameter can be empirically estimated using historical data.

Fig. 2 depicts an illustrative diagram of the examination process for any subject.

Since the aforementioned input parameters are tuned up using the historical values, it is expected that the simulation results will be in accordance with the actual subject/grade rates. Note that the success subject ratios generally do not have significant variations from one year to another. In fact, the results obtained of the developed software have been compared with the actual observed ratios, and it can be concluded that the simulator results make sense. The historic data corresponds to the academic results in the Bachelor in Engineering Technologies at the Technical University of Madrid, Spain, using the average values considering ten years, from 2001 until 2011.

2.2. Performance indicators

The “performance indicators” are a set of metrics that evaluate the academic performance of a particular academic degree. In this paper, the performance indicators considered are the ones established by the Spanish Agency for Quality and Accreditation (hereinafter called ANECA). These indicators can be divided into two main categories: *degree indicators* and *subject indicators*. A degree indicator quantifies the performance of the degree as a whole, whereas a subject indicator evaluates the performance for a particular subject.

The simulator analyzes the following five probabilistic indicators:

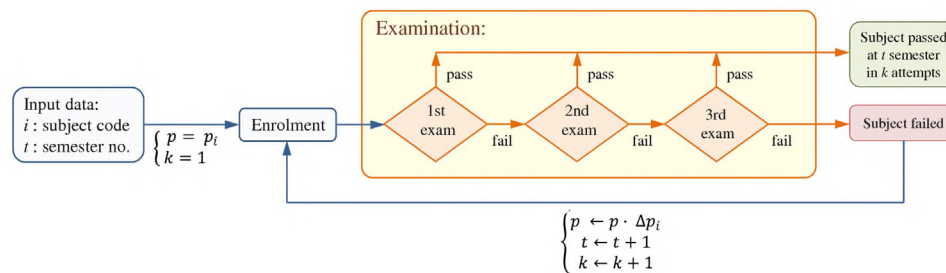


Fig. 2. Diagram of passing a subject.

2.2.1. Degree indicators

The degree indicators considered are: Degree Graduation Rate and Degree Efficiency Rate.

- **Degree Graduation Rate (DGR)** is the percentage of students who graduated as planned or taking one extra year.

DGR for the y -th year is thus computed as:

$$DGR_y [\%] = \frac{(GS_y)}{(RS_{y-n}) + (RS_{y-n-1})} \cdot 100 \quad (1)$$

where GS_y is the number of students who graduated in the y -th year, RS_y corresponds to the number of students who registered in the course at the y -th year, and n is the planned duration of the degree in years. As it can be observed from (1), the ratio DGR is computed as the graduated students divided by the number of students who started his/her studies either n or $n+1$ years before.

- **Degree Efficiency Rate (DER)** is the ratio between the credits which the students would have taken if they had passed the first time in every subject (theoretical figure) with respect to the number of credits actually taken.

DER for the y -th year is thus computed as:

$$DER_y [\%] = \frac{DC \cdot (GS_y)}{\sum_{t=1}^{n_t} \sum_{s \in GS_y} SC_{s,t}} \cdot 100 \quad (2)$$

where $SC_{s,t}$ is the number of credits that the s -th student has registered for during the t -th semester, and the constant DC corresponds to the required number of credits of the corresponding academic degree. For the case under consideration, $DC = 240$ ECTS credits. As it can be observed from (2), the ratio DER is computed as the theoretic number of credits divided by the actual number of attended credits.

2.2.2. Subject indicators

The subject indicators considered are: Subject Efficiency Rate, Subject Success Rate, and Subject Absentism Rate.

- **Subject Efficiency Rate (SER_{*i,t*})** is the percentage of students who have passed the i -th subject with respect to those who registered for it.

SER_{*i,t*} for the t -th semester is thus computed as:

$$SER_{i,t} [\%] = \frac{(AS_{i,t})}{(RS_{i,t})} \cdot 100 \quad (3)$$

where $AS_{i,t}$ is the number of students who passed the i -th subject in the t -th semester, and $RS_{i,t}$ corresponds to the number of students registered for the i -th subject at the t -th semester. As it can be observed from (3), the ratio $SER_{i,t}$ is computed as the number of successful students divided by the number of registered students, for the i -th subject at the t -th semester.

- **Subject Success Rate (SSR_{*i,t*})** is the percentage of students who passed the i -th subject. For the computation of this ratio, those students who were enrolled but did not attend to the *subject evaluation process* are disregarded. Note that the evaluation process is the exam or test used to quantify the student performance for a determined subject. SSR_{*i,t*} for the t -th semester is thus computed as:

$$SSR_{i,t} [\%] = \frac{(AS_{i,t})}{(ES_{i,t})} \cdot 100 \quad (4)$$

where $ES_{i,t}$ is the number of students who took the evaluation exam at the t -th semester for the i -th subject, and $AS_{i,t}$ corresponds to the number of students who passed it at the t -th semester for the i -th subject.

As it can be observed from (4), the ratio $SSR_{i,t}$ is computed as the number of successful students divided by the number of students who attended to the evaluation exam, for the i -th subject at the t -th semester.

- **Subject Absentism Rate (SAR_{*i,t*})** is the percentage of students who have not taken the exam of the i -th subject with respect to those who registered for the course.

SAR_{*i,t*} for the t -th semester is thus computed as:

$$SAR_{i,t} [\%] = \frac{(RS_{i,t}) - (ES_{i,t})}{(RS_{i,t})} \cdot 100 \quad (5)$$

where $RS_{i,t}$ is the number of students registered for at the i -th subject at the t -th semester, and $ES_{i,t}$ is the number of students who took the evaluation exam at the t -th semester for the i -th subject.

As it can be observed from (5), the ratio $SAR_{i,t}$ is computed as the number of students who did not attend to the evaluation exam divided by the number of registered students, for the i -th subject at the t -th semester.

The aforementioned indicators are the ones defined by the National Quality Evaluation and Accreditation Agency (ANECA, 2013), applied by all the Spanish Universities. The proposed software can be easily extended to any foreign University by appropriately redefining the indicators according to the evaluation agency of the corresponding country/state.

2.3. Background assumptions

The stochastic model is based on the following assumptions:

1. There is independence between the academic performance of different students, i.e., the passing probability p_i of student s at the subject i is not influenced whether or not the student s' has passed it (or any other subject).
2. For a given student s , future academic performance is independent of his/her past performance, i.e., the number of attempts before passing for subject i (k_i) is not directly influenced by the number of attempts of any other subject i' ($\text{corr}(k_i, k_{i'}) = 0; \forall i \neq i'$). On the other hand, a failed subject can prevent the future enrollment in the next semester for determined subjects. In this sense, the past student grades may affect the future academic performance.
3. The probability of passing the subject i at the k -th attempt is lower than the probability of passing the same subject at the $k+1$ attempt (in case of failing). This increment of probability is defined as Δp (see Fig. 2).
4. It is assumed that the student s will register at the semester $t+1$ for: (i) those subjects failed in the previous semester t , and (ii) a set of new subjects, in such a way that the number of credits does not surpass the maximum allowed.
5. The passing probability of each subject (p_i) is modeled as a uniformly-distributed random variable defined in the range $[p_i^{\text{low}}, p_i^{\text{up}}]$. This range is determined based on historic data and computed as follows:

$$[p_i^{\text{low}}, p_i^{\text{up}}] = p_i^{\text{historic}} \pm z_{\alpha/2} \cdot \sqrt{\frac{p_i^{\text{historic}} \cdot (1 - p_i^{\text{historic}})}{\#students}} \quad (6)$$

where p_i^{historic} corresponds to the passing probability for the i -th subject based on historic data; $z_{\alpha/2}$ stands for the inverse cumulative distribution of the standardized Gaussian distribution evaluated at α ; and $\#students$ is the average number of students for the i -th subject. Equation (6) is a standard confidence interval based on the binomial distribution.

The developed software is general, since any academic degree can be easily implemented by appropriately adjusting the required parameters: number of subjects, number of semesters, maximum number of allowed credits, University regulations, etc. Additionally, the historical data concerning the curricula of past students is required to appropriately tune up the pass/fail probabilities. If the simulator is extended to any other academic degree, the aforementioned assumptions should be revisited and/or modified according to the University regulation considered.

2.4. Monte Carlo method

Monte Carlo (Rubinstein, 1981) method was originally developed in the context of neutron physics (Metropolis et al., 1953) and has since then been applied to a very wide range of problems in Mathematics, Natural and Behavioral Sciences.

The Monte Carlo method is a type of computational algorithm which relies on repeated random sampling. Because of the simplicity associated to the repeated computation of random numbers, it is generally used when computing the exact result with a deterministic algorithm is cumbersome or even unfeasible. For problems with a reduced number of input parameters, this method is less competitive; but as the dimension grows it becomes the best or even the unique solution. For the problem under consideration, note that the number of input parameters is excessively high to address it from an analytical perspective.

The Monte Carlo method employed is described below:

- Step 1) Initialization of counters: student counter ($s \leftarrow 1$), year counter ($y \leftarrow 1$) and semester counter ($t \leftarrow 1$).
Initialization of sets: $PS_s = FS_s = \emptyset$ as the sets of passed subjects and failed subjects for all students, respectively.
Initialization of passing probabilities: for each subject, the passing probability p_i is randomly determined within bound defined by $[p_i^{\text{low}}, p_i^{\text{up}}]$ in (6).
- Step 2) Enrollment of the s -th student. First, the student is enrolled in those failed subjects of set FS_s . Then, the student is enrolled in the next subjects according to the Study Plan, never surpassing the maximum number of credits allowed (MCA).
Update the set of registered students $RS_{i,t}$.
- Step 3) Evaluation. The s -th student is stochastically evaluated according to the passing probability of the corresponding subjects, considering the three exams (see Fig. 1).
The sets of passed and failed subjects are updated (PS_s and FS_s , respectively).
- Step 4) Update counter $s \leftarrow s+1$. If $s \leq 450$, then go to Step 2). Otherwise, set $s \leftarrow 1$ and continue.
- Step 5) Compute the sets of presented and success students for the t -th semester ($PS_{i,t}$ and $AS_{i,t}$, respectively).
Update the semester counter: $t \leftarrow t+1$. If $t \leq 2$, then go to Step 2). Otherwise, set $t \leftarrow 1$ and continue.
Update the year counter: $y \leftarrow y+1$, and continue.
- Step 6) If $FS_s = \emptyset$, $\forall s$ and the set of passed subjects for all students (PS_s) comply the Academic Planning, the algorithm finished. Otherwise, go to Step 2).

The algorithm above has been implemented in Matlab without loss of generality, i.e, any other Study Plan can be straightforwardly implemented.

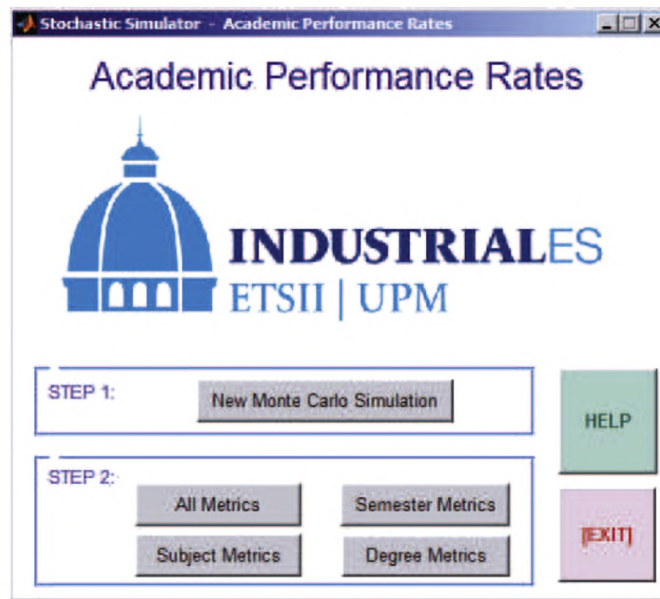


Fig. 3. Main screen of the simulator.

2.5. Software developed

The software developed has been implemented in Matlab and the interactive interface has been created using the *Matlab Graphical User Interface Development Environment* (usually known as GUIDE).

The simulator uses a user-friendly dynamic graphical interface and provides an instant visual feedback leading to an intuitive understanding of results.

The main page of the program is shown in Fig. 3. From this screen it is possible to proceed to new simulations ("New Monte Carlo Simulation" button) and to calculate the different performance indicators. Additionally, there exists a Tutorial ("Help" button) where information about the program and the performance indicators is included.

The rest of the functions are briefly explained:

2.5.1. All metrics

It provides a joint representation of all the subject metrics for a given semester or for a given track.

2.5.2. Subject metrics

Once a subject has been selected from the list, the function displays the time evolution of the performance indicators for the given subject: efficiency rate, success rate and absentism rate.

2.5.3. Degree metrics

The performance indicators of the academic degree are displayed. Namely, the time evolution of the efficiency rate and the graduation rate are provided.

2.5.4. Semester metrics

Once a semester in the list has been selected, the function provides the time evolution of the performance indicators for those subjects scheduled in the given semester.

2.6. Software applications

The simulator is designed as a decision-making tool, based on a Monte Carlo simulation which employs actual data. The software can be used for analyzing the actual/future academic performance of students, helping to make decisions such as:

- *Identification of bottleneck subjects*: The developed software allows identifying those subjects with the highest influence over the required number of years for graduation. The identification of these bottlenecks is of great interest to the Academic Committee.
- *Classroom dimension*: Based on the simulation results, the software computes the number of enrolled students (both first-time students and repeat students) for each subject. This information is of great interest to the Academic Committee for deciding the number of groups, dimension of classrooms, distribution of resources, etc.
- *Adjustment of the "maximum credits allowed"*: Studying the results of the sensitivity analysis, one may observe the influence over the adjustments of parameter MCA over the efficiency and graduation rates. Thus, if the Academic Committee is planning a readjustment of MCA, the software can be used to "predict" the expected values for the aforementioned rates.

Table 1
Factors and their levels.

	Names	Nomenclature	Levels
Factor	Maximum credits allowed (MCA)	α_i	36 40
	Increment of probability (Δp)	β_j	Low (30%) Medium (50%) High (80%)
	Range of probability (α)	γ_k	Medium (0.05%) High (0.01%)

- **Adjustment of parameter Δp :** The sensitivity analysis allows to observe the influence of the parameter Δp over the efficiency and graduation rates. The parameter p represents the academic improvement for a repeat student who is attending again to the evaluation exam. Thus, this parameter can be modified by: teaching extra classes for repeat students, student coaching, supervising the work of repeat students, etc.

3. Sensitivity analysis

In this section, a sensitivity analysis is performed, which estimates the effect on the performance indicators (as summarized in Section 2.2) of changes in the so-called *sensitivity parameters* (defined in Section 2.1) (Saltelli, Chan, & Scott, 2009).

Specifically, the sensitivity parameters considered in this study are:

Maximum credits allowed is the maximum number of credits that each student is allowed to register for per semester. According to the academic regulations of the ETSII-UPM, this limit is established in 36 credits. The alternative value considered is 40 credits.

Increment of probability is the average increase of the passing probability for a student who is examined for a failed subject. Three different values of Δp are considered: low (30%), medium (50%), and high (80%).

Range of probability is the range of the passing probability for a given subject ($[p_i^{\text{low}}, p_i^{\text{up}}]$) which is computed based on a given confidence level α . Two typical values of α are considered in this analysis: 0.05 and 0.01.

The performance metrics (outputs) considered in the sensitivity analysis are the two above-mentioned degree indicators: degree graduation rate and degree efficiency rate.

3.1. Design of experiments

In order to check if the variation of the sensitivity parameters produces a significant change of the performance metrics, a Design of Experiments (DoE) is performed and its results analyzed.

The model used in this DoE comprises the three above mentioned factors: “Maximum credits allowed”, “Increment of probability”, and “Range of probability”, as shown in Table 1. The last column provides the levels for each factor.

According to Table 1, the number of required scenarios is $2 \times 3 \times 2 = 12$. Thus, the stochastic simulator described in Section 2.4 should be run 12 times, varying the values of factors MCA, Δp , and α . The flow diagram of the stochastic simulator algorithm is depicted in Fig. 4. Note that steps 3 and 4 (students’ enrollment and examination) are conditional on the values of the factors MCA, Δp , and α .

Table 2 provides the values for the three factors considered: MCA, Δp , and α , for each scenario. These scenarios can be straightforwardly programmed using three FOR loops, as shown in the schematic code in Fig. 5. Note that indices i , j , and k correspond to the different values for factors MCA, Δp , and α , respectively.

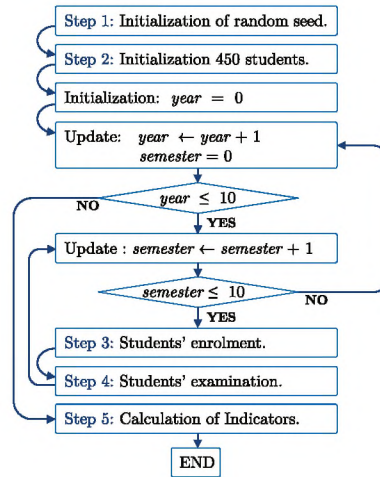


Fig. 4. Flow diagram of the stochastic simulator.

Table 2
Values employed for each scenario.

	Values		
Scenario 1	MCA = 36	$\Delta p = 0.3$	$\alpha = 0.05$
Scenario 2	MCA = 36	$\Delta p = 0.3$	$\alpha = 0.01$
Scenario 3	MCA = 36	$\Delta p = 0.5$	$\alpha = 0.05$
Scenario 4	MCA = 36	$\Delta p = 0.5$	$\alpha = 0.01$
Scenario 5	MCA = 36	$\Delta p = 0.8$	$\alpha = 0.05$
Scenario 6	MCA = 36	$\Delta p = 0.8$	$\alpha = 0.01$
Scenario 7	MCA = 40	$\Delta p = 0.3$	$\alpha = 0.05$
Scenario 8	MCA = 40	$\Delta p = 0.3$	$\alpha = 0.01$
Scenario 9	MCA = 40	$\Delta p = 0.5$	$\alpha = 0.05$
Scenario 10	MCA = 40	$\Delta p = 0.5$	$\alpha = 0.01$
Scenario 11	MCA = 40	$\Delta p = 0.8$	$\alpha = 0.05$
Scenario 12	MCA = 40	$\Delta p = 0.8$	$\alpha = 0.01$

For the sake of statistical robustness, one hundred simulations are used ($l = 1, \dots, 100$), varying the random seed. The response of each simulation y_{ijkl} is implemented as a three-way DoE model considering second-order and third-order interactions (Kuehl, 2000; Scheffé, 1959):

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + u_{ijkl}$$

where $\sum \alpha_i = \sum \beta_j = \sum \gamma_k = \sum (\alpha\beta)_{ij} = \sum (\alpha\gamma)_{ik} = \sum (\beta\gamma)_{jk} = \sum (\alpha\beta\gamma)_{ijk} = 0$, and $u_{ijkl} \sim N(0, \sigma^2)$.

Parameter μ is the global effect, i.e., the average value of the response variable. Parameter α_i is the main effect of the “Maximum Credits Allowed”, and quantifies the increase/decrease of the average of the response variable caused by a variation of the maximum number of allowed credits. Similarly, parameter β_j is the main effect of “Increment of Probability” (Δp). Likewise, parameter γ_k is the main effect of “Range of probability” (α).

Second-order interactions $\{(\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}\}$ quantify the influence on the response variable by a variation of two factors simultaneously (MCA- Δp , MCA- α , and Δp - α , respectively). The third-order interaction $((\alpha\beta\gamma)_{ijk})$ quantifies the influence over the response variable by a variation of three factors simultaneously (MCA- Δp - α). Finally, the error term u_{ijkl} includes the effects of all other causes not modeled, in our case the nearly negligible Monte Carlo error. This term is assumed to follow an independent and identically distributed process as a $N(0, \sigma^2)$.

Once the model has been estimated and the aforementioned parameters have been computed, the total sum of squares (corresponding to the total variability) can be decomposed (Kuehl, 2000; Scheffé, 1959) as:

$$SS_{total} = SS_{explained} + SS_{unexplained} = SS_{\alpha} + SS_{\beta} + SS_{\gamma} + SS_{\alpha\beta} + SS_{\alpha\gamma} + SS_{\beta\gamma} + SS_{\alpha\beta\gamma} + SS_{residuals} \quad (7)$$

The value of each term quantifies the contribution of the effect or interaction to output variability.

3.2. Degree efficiency rate: results

The first sensitivity analysis is performed to study the degree efficiency rate, using a set of one hundred simulations. Within each simulation, a cohort of 450 students is sampled, and the passing probabilities changes along simulations, generated at random from a uniform distribution in the range given by the confidence interval. The sum of squares decomposition is provided in Table 3.

```

For  $i = \{1, 2\}$ 
  For  $j = \{1, 2, 3\}$ 
    For  $k = \{1, 2\}$ 
      MCA =  $i$ -th element in  $\{36 \ 40\}$ 
       $\Delta p$  =  $j$ -th element in  $\{0.3 \ 0.5 \ 0.8\}$ 
       $\alpha$  =  $k$ -th element in  $\{0.05 \ 0.01\}$ 
      Step 1: Initialization of random seed.
      Step 2: Initialization 450 students.
      For  $year = \{1, \dots, 10\}$ 
        For  $semester = \{1, 2\}$ 
          Step 3: Students' enrolment.
          Step 4: Students' examination.
        End
      End
      Step 5: Calculation of Indicators
    End
  End
End

```

Fig. 5. Schematic code of the sensitivity analysis applied to the stochastic simulator.

Table 3

Sum of squares decomposition for the degree efficiency ratio.

Source	Sum of squares
Δp	3263.02
MCA	51.49
α	2.92
$\Delta p^* \text{MCA}$	3.45
$\Delta p^* \alpha$	0.26
$\text{MCA}^* \alpha$	0.03
$\Delta p^* \text{MCA}^* \alpha$	0.41
Residual	240.75
Total	3562.33

From Table 3, it is observed that:

- The term corresponding to the main effect “Increment of probability” is significantly higher than the rest (more than one order of magnitude). Thus, a variation of parameter Δp will lead to a significant increment/decrement of the efficiency rate.
- The second higher value is the term corresponding to “MCA”. Then, a variation of the maximum number of credits allowed will moderately vary the efficiency rate.

In order to compare the variations of the degree efficiency rate (DER), Fig. 6 provides the average value of the DER for each Δp and MCA, and its confidence interval (denoted with small horizontal lines). Additionally, Fig. 7 shows the histograms for these values of DER: the first, second and third rows of plots correspond to $\Delta p = 30\%$, $\Delta p = 50\%$ and $\Delta p = 80\%$, respectively. The first and second columns of plots correspond to MCA equal to 36 and 40, respectively.

From Figs. 6 and 7 it is observed:

- An increment of MCA (from 36 to 40) produces an average increase of 0.5% in the degree efficiency rate, averaged over the remaining factors.
- An increment of Δp (from 30% to 50%, and from 50% to 80%) produces an average increase of 2.2% and 1.7% in the DER, respectively.
- As observed from Table 3, the influence of the parameter Δp is higher than the influence of MCA.
- The variability shown in Fig. 7 is due to the randomness of the student paths, caused by the random passing/failing events in the evaluation exams along the degree.

3.3. Degree graduation rate: results

The second sensitivity analysis is carried out for the metric “degree graduation rate” (DGR), and the sum of squares decomposition is shown in Table 4.

From Table 4, it is observed that:

- The terms corresponding to the main effects “Maximum credits allowed” and “Increment of probability” are highest. Thus, a variation of either parameter MCA or Δp is expected to produce a significant increment/decrement in the graduation rate.

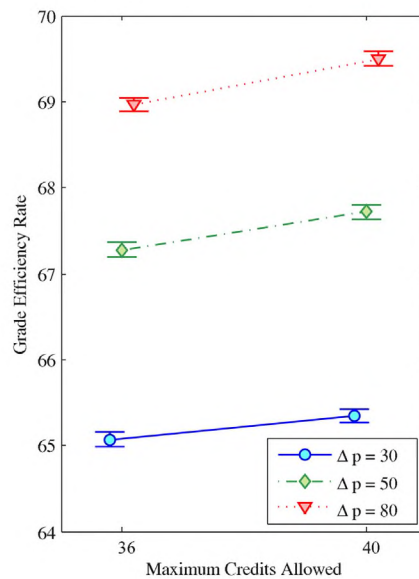


Fig. 6. Degree efficiency rate: sensitivity analysis.

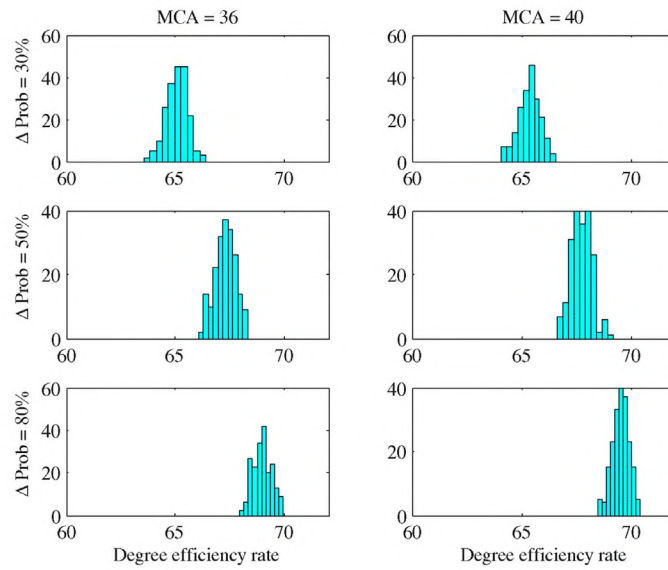


Fig. 7. Degree efficiency rate: histogram.

Table 4
Sum of squares decomposition.

Source	Sum of squares
Δp	171,335.8
MCA	510,697.9
α	172.6
$\Delta p * MCA$	2805.8
$\Delta p * \alpha$	17.9
$MCA * \alpha$	0.5
$\Delta p * MCA * \alpha$	35.9
Residual	12,344.4
Total	697,410.9

- In this case, the term corresponding to MCA is higher than the term of Δp . This will imply that the influence of the parameter MCA will be higher.

As in the previous subsection, to compare the variations of the degree graduation rate (DGR), [Fig. 8](#) provides the average value of the DGR for each Δp and MCA, and its confidence interval; and [Fig. 7](#) shows the histograms for these values of DGR.

From [Figs. 8 and 9](#) it is observed:

- An increment of MCA (from 36 to 40) produces an average increase of 40% in the graduation rate, averaged over the other two factors.
- An increment of Δp (from 30% to 50%, and from 50% to 80%) produces an average increase of 16% and 15% in the DGR, for MCA = 36. If MCA = 40, these increments are 19% and 7%, respectively.
- In this case, note that the influence of the parameter MCA is higher than that of Δp , as expected from [Table 4](#).

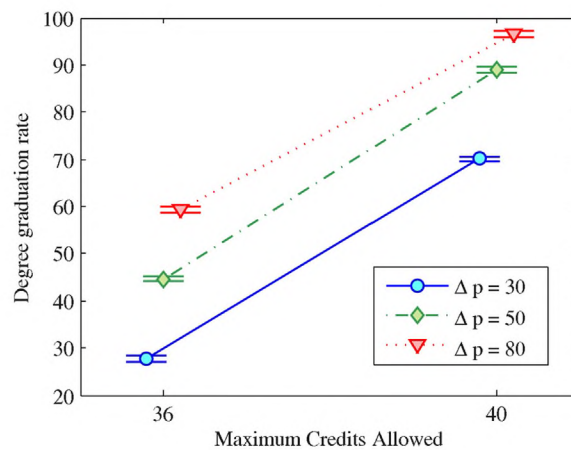


Fig. 8. Degree graduation rate: sensitivity analysis.

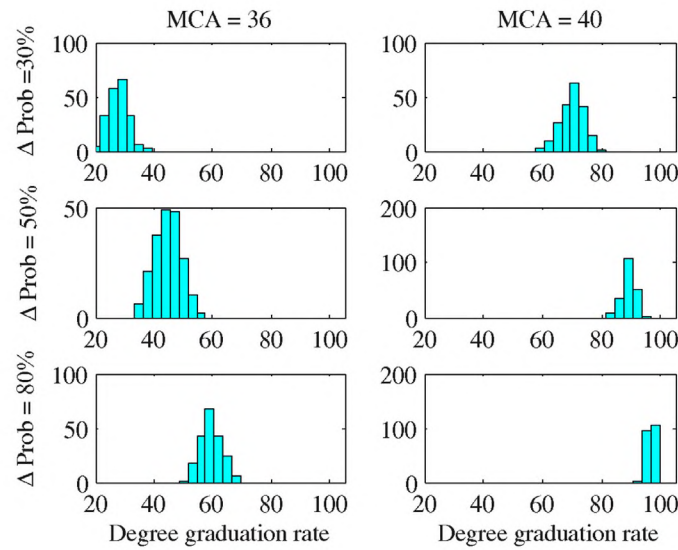


Fig. 9. Degree graduation rate: histogram.

Table 5

Comparison between measured historical metrics and the results obtained from the first-year subjects in the 2010–2011 academic year.

Year	Semester	Subject code	Measured results (historical data)		Obtained results (stochastic simulator)	
			SSR _i	SER _i	Interval for SSR _i	Interval for SER _i
First year	First semester	S_1026	57%	51%	[44%–64%]	[40%–56%]
		S_1031	76%	71%	[61%–82%]	[55%–72%]
		S_1032	71%	63%	[64%–82%]	[59%–71%]
		S_1033	61%	58%	[45%–62%]	[40%–60%]
		S_1034	87%	87%	[77%–97%]	[71%–88%]
	Second semester	S_1035	57%	53%	[38%–55%]	[33%–54%]
		S_1036	59%	52%	[46%–60%]	[41%–54%]
		S_1041	53%	46%	[37%–57%]	[33%–50%]
		S_1042	89%	85%	[75%–93%]	[70%–84%]
		S_1043	73%	58%	[61%–79%]	[56%–69%]

3.4. Comparison between historical metrics and results obtained

In order to check the validity of the results obtained using the stochastic-based software, in this section we compare some historical data and the results obtained from the stochastic simulator. Table 5 provides the success rate (SSR_i) and the efficiency rate (SER_i) for the ten first-year subjects (first and second semester) in the 2010–2011 academic year. These values (fourth and fifth columns) correspond to the historical data provided by the Secretary's Office. For confidentiality reasons, subject names are intentionally omitted.

Two hundred simulations have been performed using the developed program. The sixth and seventh columns from Table 5 provide the 1% and 99% quantiles of the results obtained, for the success rate (SSR) and the efficiency rate (SER) for each subject.

From Table 5 it can be observed that the results from the stochastic simulator are in accordance with the historical data, for the first-year subjects in the 2010–2011 academic year. Please, note that nine out of ten theoretical values (fourth and fifth columns) are included in the interval provided by the stochastic simulator (sixth and seventh columns).

3.5. Case study conclusions

Analyzing the results of the previous sections, it can be concluded:

- The most influential factor for the Degree Graduation Rate is the maximum number of credits allowed. Simulations show that an increment of 4 credits for parameter MCA ($MCA^{new} = MCA^{old} + 4$ credits) will result on an expected increment of 40 of the DGR ($DGR^{new} \approx DGR^{old} + 40$).
- On the other hand, the Degree Efficiency Rate is mainly influenced by the parameter Δp . Thus, if the Academic Committee is planning to increase this indicator, the following actions should be considered: teaching extra classes for repeat students, student coaching, supervising the work of repeat students, etc.
- The subject "Subject_0305"¹ has been identified as having the highest influence over the indicator DGR (i.e., bottleneck subject). Thus, if the Academic Committee is planning to improve this rate, the aforementioned subject should be examined in detail (evaluation, professors, subject curricula, etc).

¹ For confidentiality reasons, the actual name of the subjects has been intentionally omitted.

- One of the subjects with highest number of enrolled students is “Subject_0205”, with a quantity of students around 800. The software can simulate the behavior of this quantity under different scenarios. For example, it is observed that, if the parameter Δp is increased ($\Delta p^{new} = \Delta p^{old} + 0.3$), the expected number of students is reduced to 600. This information is of great interest to the Academic Committee, and can be used to decide the number of classrooms, number of professors, optimize resources, etc.

4. Conclusions

A computer-based tool is developed in this paper, which allows to analyze the main features of any Academic Degree, such as the efficiency, graduation rate or absenteeism. Specifically, the Bachelor of Engineering Technologies of the Technical University of Madrid has been implemented.

Using historical data from the Office of the University Secretary, the program can simulate the stochastic academic performance of the students' paths, based on a Monte Carlo routine. Additionally, a statistical procedure (Design of Experiments) is used to determine which are the parameters whose variation can modify more significantly the performance indicators.

Analyzing the results, it has been observed that the indicator DER is most influenced by the parameter Δp . On the other hand, indicator DGR is most influenced by the parameter MCA. Thus, if the Academic Committee is planning to improve these indicators, these parameters have to be examined. The software can also be used for the detection of bottleneck subjects. Specifically, for the academic degree considered, the subject “Subject_0205” has been identified as a subject with higher influence over the DGR. Another application of the developed software is to estimate the expected number of enrolled students in any subject for different scenarios (varying parameter MCA, Δp , etc); this information can be used to decide the number of classrooms, number of professors, optimize resources, etc.

Future research will focus on: (i) implementation of a dependency structure between the grades of subjects within the same knowledge area; (ii) model of different parameters Δp_i , depending on the considered subject; and (iii) extension of the sensitivity analysis to other parameters (e.g., passing probability of subjects).

The simulator is proved to be a very useful decision-making tool, particularly to decide the changes that could be made in the University to increase success rates and/or optimize resources.

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Appendix A. Nomenclature

$AS_{i,t}$	Set of students who passed the evaluation exam at the t -th semester for the i -th subject.
$RS_{i,t}$	Set of students registered at the i -th subject at the t -th semester.
$SC_{s,t}$	Number of credits that the s -th student is registered during the t -th semester.
GS_y	Set of students who graduated in the y -th year.
RS_y	Set of students who registered in the course at the y -th year.
DGR	Degree Graduation Rate.
DER	Degree Efficiency Rate.
SER_i	Subject Efficiency Rate for the i -th subject.
SSR_i	Subject Success Rate for the i -th subject.
SAR_i	Subject Absentism Rate for the i -th subject.
k_i	Number of attempts needed for passing subject i .
p_i	Probability of passing the subject i .
Δp	Average increment of passing probability for a failed course.

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